Integral Table

Basic Properties of Integration

$$\int f(x)dx = F(x) + C\,, \quad \text{if } F'(x) = f(x)$$

$$\left[\int f(x)dx\right]' = f(x)$$

$$\int af(x)dx = a\int f(x)dx$$

$$\int\left[f(x) \pm g(x)\right]dx = \int f(x)dx \pm \int g(x)dx$$

$$\int f(ax)dx = \frac{1}{a}F(ax) + C$$

$$\int f(a+bx)dx = \frac{1}{b}F(a+bx) + C$$

$$\int f(x)f'(x)dx = \frac{1}{2}f^2(x) + C$$

$$\int \frac{f'(x)}{f(x)}dx = \ln\left|f(x)\right| + C$$

$$\int ydx = xy - \int xdy$$

Common Integrals

45. Miscellaneous

Simple Functions

$$\int dx = x + C$$

$$\int a dx = ax + C$$

$$\int x dx = \frac{1}{2}x^2 + C$$

Powers of *x*

$$\int x^2 dx = \frac{1}{3}x^3 + C$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + C, \quad n \neq -1$$

Inverse Powers of *x*

$$\int \frac{1}{x} dx = \ln|x| + C, \quad x \neq 0$$

$$\int \frac{1}{x^n} dx = \frac{1}{1-n}x^{1-n} + C, \quad n \neq 1$$

$$\int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + C$$

Exponentials

$$\int e^x dx = e^x + C$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + C$$

Powers *and* Exponentials

$$\int xe^{x}dx = (x-1)e^{x} + C$$

$$\int xe^{ax}dx = \frac{1}{a^2}e^{ax}(ax-1) + C$$

$$\int x^2e^{x}dx = (x^2-2x+2)e^{x} + C$$

$$\int x^2e^{ax}dx = \left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right)e^{ax} + C$$

$$\int x^ne^{x}dx = x^ne^{x} - n\int x^{n-1}e^{x}dx + C$$

$$\int x^ne^{ax}dx = \frac{1}{a}\left(x^ne^{ax} - n\int x^{n-1}e^{ax}dx\right) + C$$

Logarithms

$$\int \ln x dx = x\ln|x| - x + C$$

$$\int \log_a x dx = \int \frac{\ln x}{\ln a}dx = \frac{x\ln x - x}{\ln a} + C$$

$$\int \ln ax dx = x\ln ax - x + C$$

$$\int \ln(ax+b)dx = \frac{ax+b}{a}\ln(ax+b) - x + C$$

$$\int (\ln|x|)^2 dx = x(\ln|x|)^2 - 2x\ln|x| + 2x + C$$

$$\int (\ln x)^n dx = x(\ln x)^n - n\int (\ln x)^{n-1} dx + C$$

Reciprocals *with* Logarithms

$$\int \frac{\ln(ax)}{x}dx = \frac{1}{2}\left[\ln(ax)\right]^2 + C$$

$$\int \frac{\ln^2 x}{x} dx = \frac{\ln^3 x}{3} + C$$

$$\int \frac{1}{x \ln x} dx = \ln \left| \ln x \right| + C$$

Powers *and* Logarithms

$$\int x \ln(ax+b) dx = \frac{b}{2a} x - \frac{1}{4} x^2 + \frac{1}{2}\left(x^2 - \frac{b^2}{a^2} \right) \ln(ax+b) + C$$

$$\int x^n \ln x dx = \frac{1}{n+1} x^{n+1} \ln x - \frac{1}{(n+1)^2} x^{n+1} + C, \quad n \neq -1$$

$$\int x^n \ln(ax) dx = \frac{1}{n+1} x^{n+1} \ln(ax) - \frac{1}{(n+1)^2} x^{n+1} + C, \quad n \neq -1$$

Trigonometric Functions: sine

$$\int \sin x dx = -\cos x + C$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax + C$$

$$\int \sin^2 x dx = \frac{1}{2} x - \frac{1}{4} \sin(2x) + C$$

$$\int \sin^2 ax dx = \frac{1}{2} x - \frac{1}{4a} \sin 2ax + C$$

$$\int \sin^3 x dx = -\frac{3}{4} \cos x + \frac{1}{12} \cos(3x) + C$$

$$\int \sin^n x dx = -\frac{1}{n}\left[\sin^{n-1} x \cos x - (n-1) \int \sin^{n-2} x dx \right] + C$$

Trigonometric Functions: cosine

$$\int \cos x dx = \sin x + C$$

$$\int \cos ax dx = \frac{1}{a}\sin ax + C$$

$$\int \cos(ax+b)\, dx = \frac{1}{a}\sin(ax+b) + C$$

$$\int \cos^2 x dx = \frac{1}{2}x + \frac{1}{4}\sin 2x + C$$

$$\int \cos^2 ax dx = \frac{1}{2}x + \frac{1}{4a}\sin 2ax + C$$

$$\int \cos^3 x dx = \sin x - \frac{1}{3}\sin^3 x + C$$

$$\int \cos^n x dx = \frac{1}{n}\left[\cos^{n-1} x \sin x + (n-1)\int \cos^{n-2} x dx\right] + C$$

Trigonometric Functions: tangent

$$\int \tan x dx = \ln|\sec x| + C = -\ln|\cos x| + C$$

$$\int \tan ax dx = -\frac{1}{a}\ln(\cos ax) + C$$

$$\int \tan^2 x dx = \tan x - x + C$$

$$\int \tan^2 ax dx = -x + \frac{1}{a}\tan ax + C$$

$$\int \tan^3 x dx = \ln(\cos x) + \frac{1}{2}\sec^2 x + C$$

$$\int \tan^n x dx = \frac{1}{n-1}\tan^{n-1} x - \int \tan^{n-2} x dx + C$$

Trigonometric Functions: cotangent

$$\int \cot x dx = \ln|\sin x| + C = -\ln|\csc x| + C$$

$$\int \cot ax dx = \frac{1}{a}\ln(\sin ax) + C$$

$$\int \cot^2 x dx = -\cot x - x + C$$

$$\int \cot^2 ax dx = -x - \frac{1}{a}\cot ax + C$$

$$\int \cot^n x dx = -\frac{1}{n-1}\cot^{n-1} x - \int \cot^{n-2} x dx + C$$

Trigonometric Functions: secant

$$\int \sec x dx = \ln|\sec x + \tan x| + C = \ln \tan\left(\frac{x}{2} + \frac{\pi}{4}\right) + C = 2\tanh^{-1}\left(\tan\frac{x}{2}\right) + C$$

$$\int \sec^2 x dx = \tan x + C$$

$$\int \sec ax dx = \frac{1}{a}\ln(\sec ax + \tan ax) + C$$

$$\int \sec^2 ax dx = \frac{1}{a}\tan ax + C$$

$$\int \sec^n x dx = \frac{1}{n-1}\left[\sec^{n-2} x \tan x + (n-2)\int \sec^{n-2} x dx\right] + C$$

Trigonometric Functions: cosecant

$$\int \csc x dx = \ln|\csc x - \cot x| + C = -\ln|\csc x + \cot x| + C = \ln\left|\tan\frac{x}{2}\right| + C$$

$$\int \csc ax dx = \frac{1}{a}\ln(\csc ax - \cot ax) + C$$

$$\int \csc^2 x dx = -\cot x + C$$

$$\int \csc^2 ax\,dx = -\frac{1}{a}\cot ax + C$$

$$\int \csc^3 x\,dx = -\frac{1}{2}\cot x \csc x + \frac{1}{2}\ln\left|\csc x - \cot x\right| + C$$

$$\int \csc^n x\,dx = -\frac{1}{n-1}\left[\csc^{n-2} x \cot x - (n-2)\int \csc^{n-2} x\,dx\right] + C$$

Products of Trigonometric Functions

$$\int \sin ax \sin bx\,dx = \frac{1}{2}\left[\frac{1}{(a-b)}\sin(a-b)x - \frac{1}{(a+b)}\sin(a+b)x\right] + C, \quad a \neq \pm b$$

$$\int \frac{1}{\sin^2 ax}dx = -\frac{1}{a}\cot ax + C$$

$$\int \sin x \cos x\,dx = \frac{1}{2}\sin^2 x + C$$

$$\int \sin ax \cos bx\,dx = -\frac{1}{2}\left[\frac{1}{(a-b)}\cos(a-b)x + \frac{1}{(a+b)}\cos(a+b)x\right] + C, \quad a \neq \pm b$$

$$\int \sin^m x \cos^n x\,dx = \frac{1}{m+n}\left[\sin^{m+1} x \cos^{n-1} x + (n-1)\int \sin^m x \cos^{n-2} x\,dx\right] + C$$
$$= -\frac{1}{m+n}\left[\sin^{m-1} x \cos^{n+1} x - (m-1)\int \sin^{m-2} x \cos^n x\,dx\right] + C$$

$$\int \cos ax \cos bx\,dx = \frac{1}{2}\left[\frac{1}{(a+b)}\sin(a+b)x + \frac{1}{(a-b)}\sin(a-b)x\right] + C, \quad a \neq \pm b$$

$$\int \sec x \tan x\,dx = \sec x + C$$

$$\int \sec x \csc x\,dx = \ln(\tan x) + C$$

$$\int \csc x \cot x\,dx = -\csc x + C$$

Reciprocals of Trigonometric Functions

$$\int \frac{1}{\sin^n x}dx = \frac{1}{(n-1)}\left[-\frac{\cos x}{\sin^{n-1} x} + (n-2)\int \frac{1}{\sin^{n-2} x}dx\right] + C, \quad n \neq 1$$

$$\int \frac{1}{\sin x \cos x}dx = \ln(\tan x) + C$$

$$\int \frac{1}{\cos^2 ax}dx = \frac{1}{a}\tan ax + C$$

$$\int \frac{1}{\cos^n x}dx = \frac{1}{(n-1)}\left[\frac{\sin x}{\cos^{n-1} x} + (n+2)\int \frac{1}{\cos^{n-2} x}dx\right] + C, \quad n \neq 1$$

Reciprocals *with* Trigonometric Functions

$$\int \frac{1}{1+\sin ax}dx = \frac{1}{a}\tan\left(\frac{\pi}{4} - \frac{a}{2}x\right) + C$$

$$\int \frac{1}{1+\cos ax}dx = \frac{1}{a}\tan\frac{a}{2}x + C$$

$$\int \frac{1}{1+\tan ax}dx = \frac{1}{2}\left[x + \frac{1}{a}\ln(\cos ax + \sin ax)\right] + C$$

$$\int \frac{1}{a\sin cx + b\cos cx}dx = \frac{1}{c\sqrt{a^2+b^2}}\ln\left\{\tan\frac{1}{2}\left[cx + \tan^{-1}\left(\frac{b}{a}\right)\right]\right\} + C$$

Powers *and* Trigonometric Functions

$$\int x\sin x dx = \sin x - x\cos x + C$$

$$\int x\sin ax dx = \frac{1}{a^2}\sin ax - \frac{1}{a}x\cos ax + C$$

$$\int x^2 \sin x dx = -x^2\cos x + 2x\sin x + 2\cos x + C$$

$$\int x^2 \sin(ax)\,dx = \frac{2}{a^2}x\sin(ax) + \frac{2-a^2x^2}{a^3}\cos(ax) + C$$

$$\int x^n \sin x\,dx = -x^n \cos x + n\int x^{n-1}\cos x\,dx + C$$

$$\int x^n \sin(ax)\,dx = -\frac{1}{a}x^n\cos(ax) + \frac{n}{a}\int x^{n-1}\cos(ax)\,dx + C$$

$$\int x\cos x\,dx = \cos x + x\sin x + C$$

$$\int x\cos ax\,dx = \frac{1}{a^2}\cos ax + \frac{1}{a}x\sin ax + C$$

$$\int x^2\cos x\,dx = x^2\sin x + 2x\cos x - 2\sin x + C$$

$$\int x^2\cos(ax)\,dx = \frac{2}{a^2}x\cos(ax) + \frac{a^2x^2-2}{a^3}\sin(ax) + C$$

$$\int x^n\cos x\,dx = x^n\sin x - n\int x^{n-1}\sin x\,dx + C$$

$$\int x^n\cos(ax)\,dx = \frac{1}{a}x^n\sin(ax) - \frac{n}{a}\int x^{n-1}\sin(ax)\,dx + C$$

Exponentials *and* Trigonometric Functions

$$\int e^x\sin x\,dx = \frac{1}{2}e^x(\sin x - \cos x) + C$$

$$\int e^{ax}\sin bx\,dx = \frac{1}{a^2+b^2}e^{ax}(a\sin bx - b\cos bx) + C$$

$$\int e^x\cos x\,dx = \frac{1}{2}e^x(\cos x + \sin x) + C$$

$$\int e^{ax}\cos bx\,dx = \frac{1}{a^2+b^2}e^{ax}(a\cos bx + b\sin bx) + C$$

Powers, Exponentials *and* Trigonometric Functions

$$\int xe^x \sin x\,dx = \frac{1}{2}e^x\left(x\sin x - x\cos x + \cos x\right) + C$$

$$\int xe^x \cos x\,dx = \frac{1}{2}e^x\left(x\sin x + x\cos x - \sin x\right) + C$$

Inverse Trigonometric Functions

$$\int \sin^{-1} x\,dx = x\sin^{-1} x + \sqrt{1-x^2} + C$$

$$\int \sin^{-1} ax\,dx = x\left(\sin^{-1} ax\right) + \frac{1}{a}\sqrt{1-a^2x^2} + C$$

$$\int \cos^{-1} x\,dx = x\cos^{-1} x - \sqrt{1-x^2} + C$$

$$\int \cos^{-1} ax\,dx = x\left(\cos^{-1} ax\right) - \frac{1}{a}\sqrt{1-a^2x^2} + C$$

$$\int \tan^{-1} x\,dx = x\tan^{-1} x - \frac{1}{2}\ln\left(x^2+1\right) + C$$

$$\int \tan^{-1} ax\,dx = x\tan^{-1} ax + \frac{1}{2a}\ln\left(1+a^2x^2\right) + C$$

$$\int \cot^{-1} x\,dx = x\cot^{-1} x + \frac{1}{2}\ln\left(x^2+1\right) + C$$

$$\int \sec^{-1} x\,dx = x\sec^{-1} x - \ln\left|x + \sqrt{x^2-1}\right| + C$$

$$\int \csc^{-1} x\,dx = x\csc^{-1} x + \ln\left|x + \sqrt{x^2-1}\right| + C$$

Powers *and* Inverse Trigonometric Functions

$$\int x\sin^{-1} x\,dx = \frac{1}{4}\left[\left(2x^2-1\right)\sin^{-1} x + x\sqrt{1-x^2}\right] + C$$

$$\int x\cos^{-1} x\,dx = \frac{1}{4}\left[\left(2x^2-1\right)\cos^{-1} x - x\sqrt{1-x^2}\right] + C$$

Hyperbolic Functions: sinh

$$\int \sinh x dx = \cosh x + C$$

$$\int \sinh ax dx = \frac{1}{a}\cosh ax + C$$

$$\int \sinh^2 x dx = \frac{1}{4}\sinh 2x - \frac{1}{2}x + C$$

Hyperbolic Functions: cosh

$$\int \cosh x dx = \sinh x + C$$

$$\int \cosh ax dx = \frac{1}{a}\sinh ax + C$$

$$\int \cosh^2 x dx = \frac{1}{4}\sinh 2x + \frac{1}{2}x + C$$

Hyperbolic Functions: tanh

$$\int \tanh x dx = \ln(\cosh x) + C$$

$$\int \tanh ax dx = \frac{1}{a}\ln(\cosh ax) + C$$

$$\int \tanh^n(ax) dx = -\frac{1}{a(n-1)}\tanh^{n-1}(ax) + \int \tanh^{n-2}(ax) dx + C, \quad n \neq 1$$

$$\int \tanh^2 x dx = x - \tanh x + C$$

Hyperbolic Functions: coth

$$\int \coth x dx = \ln(\sinh x) + C, \quad x > 0$$

$$\int \coth ax dx = \frac{1}{a}\ln\left[\sinh(ax)\right] + C$$

$$\int \coth^n(ax)\,dx = -\frac{1}{a(n-1)}\coth^{n-1}(ax) + \int \coth^{n-1}(ax)\,dx + C\,, \quad n \neq 1$$

Hyperbolic Functions: sech

$$\int \operatorname{sech} x dx = \tan^{-1}(\sinh x) + C$$

$$\int \operatorname{sech} ax dx = \frac{2}{a}\tan^{-1}\left[\tanh\left(\frac{a}{2}x\right)\right] + C$$

$$\int \operatorname{sech}^2 x dx = \tanh x + C = -\coth^{-1}(\cosh x) + C$$

Hyperbolic Functions: csch

$$\int \operatorname{csch} x dx = \ln\left|\tanh\left(\frac{1}{2}x\right)\right| + C$$

$$\int \operatorname{csch} ax dx = \frac{1}{a}\ln\left[\tanh\left(\frac{a}{2}x\right)\right] + C$$

$$\int \operatorname{csch}^2 x dx = -\coth x + C$$

Products of Hyperbolic Functions

$$\int \sinh x \cosh x dx = \frac{1}{2}\cosh^2 x + C$$

$$\int \sinh(ax)\cosh(ax)\,dx = \frac{-2ax + \sinh(2ax)}{4a} + C$$

$$\int \sinh(ax)\cosh(bx)\,dx = \frac{b\cosh(bx)\sinh(ax) - a\cosh(ax)\sinh(bx)}{b^2 - a^2} + C$$

$$\int \operatorname{sech} x \tanh x dx = -\operatorname{sech} x + C$$

$$\int \operatorname{csch} x \coth x dx = -\operatorname{csch} x + C$$

Reciprocals of Hyperbolic Functions

$$\int \frac{1}{\sinh^n(ax)} dx = \frac{\cosh(ax)}{a(n-1)\sinh^{n-1}(ax)} - \frac{n-2}{n-1}\int \frac{1}{\sinh^{n-2}(ax)} dx + C, \quad n \neq 1$$

$$\int \frac{1}{\cosh(ax)} dx = \frac{2}{a}\tan^{-1} e^{ax} + C$$

$$\int \frac{1}{\cosh^n(ax)} dx = \frac{\sinh(ax)}{a(n-1)\cosh^{n-1}(ax)} + \frac{n-2}{n-1}\int \frac{1}{\cosh^{n-2}(ax)} dx + C, \quad n \neq 1$$

Powers *and* Hyperbolic Functions

$$\int x \sinh x dx = x \cosh x - \sinh x + C$$

$$\int x \sinh(ax) dx = \frac{1}{a} x \cosh(ax) - \frac{1}{a^2}\sinh(ax) + C$$

$$\int x \cosh x dx = x \sinh x - \cosh x + C$$

$$\int x \cosh(ax) dx = \frac{1}{a} x \sinh(ax) - \frac{1}{a^2}\cosh(ax) + C$$

$$\int x^2 \cosh(ax) dx = -\frac{2x\cosh(ax)}{a^2} + \left(\frac{x^2}{a} + \frac{2}{a^3}\right)\sinh(ax) + C$$

Exponentials *and* Hyperbolic Functions

$$\int e^{ax} \sinh bx dx = \frac{1}{a^2 - b^2} e^{ax}\left(a \sinh bx - b\cosh bx\right) + C, \quad a \neq b$$

$$\int e^{ax}\cosh bx\,dx = \frac{1}{a^2-b^2}e^{ax}\left(a\cosh bx - b\sinh bx\right) + C\,,\quad a \neq b$$

Products of Trigonometric *and* Hyperbolic Functions

$$\int \sin(ax)\sinh(bx)\,dx = \frac{b\cosh(bx)\sin(ax) - a\cos(ax)\sinh(bx)}{a^2+b^2} + C$$

$$\int \sin(ax)\cosh(bx)\,dx = \frac{-a\cos(ax)\cosh(bx) + b\sin(ax)\sinh(bx)}{a^2+b^2} + C$$

$$\int \cos(ax)\cosh(bx)\,dx = \frac{a\sin(ax)\cosh(bx) + b\cos(ax)\sinh(bx)}{a^2+b^2} + C$$

$$\int \cos(ax)\sinh(bx)\,dx = \frac{b\cos(ax)\cosh(bx) + a\sin(ax)\sinh(bx)}{a^2+b^2} + C$$

Inverse Hyperbolic Functions

$$\int \sinh^{-1} ax\,dx = x\sinh^{-1} ax - \frac{\sqrt{a^2x^2+1}}{a} + C$$

$$\int \cosh^{-1} ax\,dx = x\cosh^{-1} ax - \frac{\sqrt{ax-1}\sqrt{ax+1}}{a} + C$$

$$\int \tanh^{-1} ax\,dx = x\tanh^{-1} ax + \frac{\ln\left(1-a^2x^2\right)}{2a} + C$$

$$\int \coth^{-1} ax\,dx = x\coth^{-1} ax + \frac{\ln\left(1-a^2x^2\right)}{2a} + C$$

$$\int \operatorname{sech}^{-1} ax\,dx = x\operatorname{sech}^{-1} ax - \frac{2}{a}\tan^{-1}\sqrt{\frac{1-ax}{1+ax}} + C$$

$$\int \operatorname{csch}^{-1} ax\,dx = x\operatorname{csch}^{-1} ax + \frac{1}{a}\coth^{-1}\left(\frac{\sqrt{1+a^2x^2}}{ax}\right) + C$$

Polynomials

$$\int (a+bx)^n \, dx = \frac{1}{b(n+1)}(a+bx)^{n+1} + C, \quad n \neq -1$$

Reciprocals of Polynomials

$$\int \frac{1}{a+bx} \, dx = \frac{1}{b} \ln|a+bx| + C$$

$$\int \frac{1}{(a+bx)^2} \, dx = -\frac{1}{b(a+bx)} + C$$

$$\int \frac{1}{(a+bx)^n} \, dx = -\frac{1}{(n-1)b(a+bx)^{n-1}} + C, \quad n \neq 1$$

$$\int \frac{1}{1+x^2} \, dx = \tan^{-1} x + C$$

$$\int \frac{1}{a^2+x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C, \quad a \neq 0$$

$$\int \frac{1}{1-x^2} \, dx = \tanh^{-1} x + C$$

$$\int \frac{1}{a^2-x^2} \, dx = \frac{1}{2a} \ln\left|\frac{a+x}{a-x}\right| + C = \frac{1}{a} \tanh^{-1} \frac{x}{a} + C, \quad x^2 < a^2$$

Powers *and* Reciprocals of Polynomials

$$\int \frac{x}{a+bx} \, dx = \frac{1}{b^2}\left[a+bx-a\ln|a+bx|\right] + C$$

$$\int \frac{x}{a^2+x^2} \, dx = \frac{1}{2} \ln\left(a^2+x^2\right) + C$$

$$\int \frac{x}{(a+bx)^2} \, dx = \frac{1}{b^2}\left[\frac{a}{a+bx} + \ln|a+bx|\right] + C$$

$$\int \frac{x}{(a+bx)^3}\,dx = -\frac{a+2bx}{2b^2(a+bx)^2} + C$$

$$\int \frac{x^2}{a+bx}\,dx = \frac{1}{b^3}\left[\frac{1}{2}(a+bx)^2 - 2a(a+bx) + a^2 \ln|a+bx|\right] + C$$

$$\int \frac{x^2}{a^2+x^2}\,dx = x - a\tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{x^2}{(a+bx)^2}\,dx = \frac{1}{b^3}\left[a+bx - \frac{a^2}{a+bx} - 2a\ln|a+bx|\right] + C$$

$$\int \frac{x^3}{a^2+x^2}\,dx = \frac{1}{2}\left[x^2 - a^2\ln\left(a^2+x^2\right)\right] + C$$

Reciprocals of Powers *and* Polynomials

$$\int \frac{1}{x(a+bx)}\,dx = \frac{1}{a}\ln\left|\frac{x}{a+bx}\right| + C$$

$$\int \frac{1}{x^2(a+bx)}\,dx = -\frac{1}{ax} + \frac{b}{a^2}\ln\left|\frac{a+bx}{x}\right| + C$$

$$\int \frac{1}{x\left(a^2+x^2\right)}\,dx = \frac{1}{2a^2}\ln\left(\frac{x^2}{a^2+x^2}\right) + C$$

$$\int \frac{1}{x(a+bx)^2}\,dx = \frac{1}{a(a+bx)} + \frac{1}{a^2}\ln\left|\frac{x}{a+bx}\right| + C$$

Square Roots of Polynomials

$$\int \sqrt{ax+b}\,dx = \left(\frac{2b}{3a} + \frac{2x}{3}\right)\sqrt{ax+b} + C$$

$$\int \sqrt{a^2+x^2}\,dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\ln\left|x+\sqrt{a^2+x^2}\right| + C$$

$$\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C, \quad |x| < a$$

$$\int \sqrt{x^2 - a^2}\,dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\ln\left|x + \sqrt{x^2 - a^2}\right| + C$$

Reciprocals of Square Roots of Polynomials

$$\int \frac{1}{\sqrt{ax+b}}\,dx = \frac{2\sqrt{ax+b}}{a} + C$$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C$$

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = \sinh^{-1} x + C$$

$$\int \frac{1}{\sqrt{x^2-1}}\,dx = \cosh^{-1} x + C$$

$$\int \frac{1}{\sqrt{a \ \ + x^2}}\,dx = \ln\left(x + \sqrt{a \ \ + x^2}\right) + C = \sinh^{=1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + C, \quad a > |x|$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \ln\left|x + \sqrt{x^2 - a^2}\right| + C, \quad |x| > a > 0$$

Powers *and* Square Roots of Polynomials

$$\int x\sqrt{ax+b}\,dx = \frac{2(3ax-2b)\sqrt{(ax+b)^3}}{15a^2} + C$$

$$\int x\sqrt{a^2+x^2}\,dx = \frac{1}{3}\left(a^2+x^2\right)^{\frac{3}{2}} + C$$

$$\int x\sqrt{x^2-a^2}\,dx=\frac{1}{3}\left(x^2-a^2\right)^{\frac{3}{2}}+C$$

Powers *with* Reciprocals of Square Roots of Polynomials

$$\int \frac{x}{\sqrt{a^2\pm x^2}}dx=\pm\sqrt{a^2\pm x^2}+C$$

$$\int \frac{x}{\sqrt{x^2-a^2}}\,dx=\sqrt{x^2-a^2}+C$$

$$\int \frac{x^2}{\sqrt{a^2-x^2}}\,dx=-\frac{1}{2}x\sqrt{a^2-x^2}-\frac{1}{2}a^2\tan^{-1}\left(\frac{x\sqrt{a^2-x^2}}{x^2-a^2}\right)+C$$

$$\int \frac{x^2}{\sqrt{x^2-a^2}}\,dx=\frac{1}{2}x\sqrt{x^2-a^2}+\frac{1}{2}\ln\left(x+\sqrt{x^2-a^2}\right)+C$$

$$\int \frac{x^2}{\sqrt{a^2+x^2}}\,dx=\frac{1}{2}x\sqrt{a^2+x^2}-\frac{1}{2}\ln\left(x+\sqrt{a^2+x^2}\right)+C$$

Square Roots of Polynomials *with* Reciprocals of Powers

$$\int \frac{\sqrt{ax+b}}{x}\,dx=2\sqrt{ax+b}+b\int \frac{1}{x\sqrt{ax+b}}\,dx+C$$

$$\int \frac{\sqrt{a^2+x^2}}{x}\,dx=\sqrt{a^2+x^2}-a\ln\left|\frac{a+\sqrt{a^2+x^2}}{x}\right|+C$$

$$\int \frac{\sqrt{a^2-x^2}}{x}dx=\sqrt{a^2-x^2}-a\ln\left|\frac{a+\sqrt{a^2-x^2}}{x}\right|+C$$

$$\int \frac{\sqrt{x^2-a^2}}{x}\,dx=\sqrt{x^2-a^2}-a\cos^{-1}\left(\frac{a}{x}\right)+C\,,\quad 0<a<|x|$$

Reciprocals of Powers *and* Square Roots of Polynomials

$$\int \frac{1}{x\sqrt{ax+b}}\,dx = \frac{1}{\sqrt{b}}\ln\left|\frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}}\right| + C$$

$$\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C\,, \quad |x| > a$$

$$\int \frac{1}{x\sqrt{a^2-x^2}}\,dx = -\frac{1}{a}\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right) + C\,, \quad 0 < x < a$$

$$\int \frac{1}{|x|\sqrt{a^2+x^2}}\,dx = -\frac{1}{a}\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right) + C\,, \quad x \neq 0$$

Miscellaneous

$$\int a^x\,dx = \frac{1}{\ln a}a^x + C\,, \quad a > 0\,, \quad a \neq 1$$

$$\int x^n a^x\,dx = \frac{1}{\ln a}\left(x^n a^x - n\int x^{n-1}a^x\,dx\right) + C$$

$$\int \frac{1}{a+be^{cx}}\,dx = \frac{1}{a}\left[x - \frac{1}{c}\ln\left(a+be^{cx}\right)\right] + C$$

$$\int \frac{a+e^x}{b+e^x}\,dx = \frac{1}{b}\left[ax - (a-b)\ln\left(b+e^x\right)\right] + C$$